FREQUENCY COMPARATOR

VCH-314

Hardware Operational Manual

411146.014HOM

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1 Designation

The frequency comparator VCH-314 is intended for precise instability measurements of standard frequencies 5 or 10 or 100 MHz (in combination with external Personal Computer).

The main comparator applications:
- crystal and quantum frequency standards production and checking,
- time keeping systems,
- scientific measurements.

2 Technical specifications

2.1 Input frequency: 5 or 10 or 100 MHz.
2.2 Input signals level: (0.6 – 1.2) V on 50 Ohm load.
2.3 Relative frequency difference multiplication factor – $K$: $1 \times 10^6$ or $1 \times 10^3$.
2.4 Frequency fluctuations passband – $B$: (10±3) Hz or 3 Hz* (* – performed by digital filtering) for $K=1 \times 10^6$ and (10±3) kHz for $K=1 \times 10^3$.
2.5 Relative difference of input signal’s frequency:
$$\pm 1.0 \times 10^{-8} \text{ for } K=1 \times 10^6,$$
$$\pm 1.0 \times 10^{-6} \text{ for } K=1 \times 10^3.$$ 
2.6 Parameters of output signal’s (Fx, Fxy1, Fxy2 on 50 Ohm load:
- nominal frequency:
  Fx output – 1 Hz;
  Fxy1 and Fxy2 outputs – 1 Hz for 10 Hz passband, 100 Hz for 3 Hz passband;
- polarity of pulses: positive;
- low-level voltage: not more 0.4 V;
- high-level voltage: not less 2.4 V;
2.7 Introduced by the comparator frequency instability (two-sample Allan variance)
- $\sigma_y(\tau) = \sqrt{\sigma_m^2(\tau) + \sigma_{ad}^2(\tau)}$, where: $\sigma_m(\tau)$ – main error ($\Delta f/f=0$); $\sigma_{ad}(\tau)$ – additional error (when $\Delta f/f\neq0$); $\tau$ – averaging time, from 1 s up to 500000 s.
2.8 The main error for one channel and two channels measuring mode for the case where $K=1\times10^6$, $B=3$ Hz: not more values given in table 2.1.

**Table 2.1**

<table>
<thead>
<tr>
<th>Averaging time, $\tau$</th>
<th>Main error, <em>two-sample Allan variance</em> ($K=1\times10^6$, $B=3$ Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“Two-oscillators, one-channel” mode, var2(Fxy1), var2(Fxy2)</td>
</tr>
<tr>
<td>1 s</td>
<td>8.0$\times10^{-14}$</td>
</tr>
<tr>
<td>10 s</td>
<td>2.0$\times10^{-14}$</td>
</tr>
<tr>
<td>100 s</td>
<td>3.0$\times10^{-15}$</td>
</tr>
<tr>
<td>1000 s and more</td>
<td>5.0$\times10^{-16}$</td>
</tr>
</tbody>
</table>

Notes:
1. *Introduced by the comparator main error is guaranteed after settling time according p. 2.11, when signals are connected to the inputs.*
2. *Introduced by the comparator main error for averaging time 1000 s and more is guaranteed provided that ambient temperature changes not more than 1 °C / hour.*
2.9 The main error for one channel and two channels measuring mode for the case where $K=1\times10^3$: not more 1.0$\times10^{-11}$ s/$\tau$ (but “floor” is not more 5.0$\times10^{-16}$).
2.10 The additional error ($\sigma_{\text{add}}(\tau)$): not more than 2.0$\times10^{-3}$Δ$f/f$.
2.11 Settling time:
- 2 h, for averaging times not more 100 s;
- 4 h, for averaging times more 100 s.
2.13 Power supply voltage: (198 – 242) V AC, 50; 60 Hz.
2.14 Power consumption: not more 30 V·A.
2.15 Ambient temperature: from +5 °C up to +40 °C.
2.16 Weight: not more 8 kg.
2.17 Dimensions (W×H×D): 236×140×370 mm.
3 Instrument set composition

The frequency comparator set composition is given in the table 3.1.

Table 3.1

<table>
<thead>
<tr>
<th>№</th>
<th>Model and designation</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Frequency comparator VCH-314</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>Power connecting cord</td>
<td>1</td>
</tr>
<tr>
<td>3.</td>
<td>RS-232 interface cable 685670.026</td>
<td>1</td>
</tr>
<tr>
<td>4.</td>
<td>Power splitter ZFSC-2-1W+</td>
<td>2</td>
</tr>
<tr>
<td>5.</td>
<td>RF interconnecting SMA / BNC cable 685670.154 (0.2 m)</td>
<td>4</td>
</tr>
<tr>
<td>6.</td>
<td>RF interconnecting BNC / BNC cable 685670.154-01 (1.5 m)</td>
<td>2</td>
</tr>
<tr>
<td>7.</td>
<td>Frequency comparator VCH-314 Installation disc</td>
<td>1</td>
</tr>
<tr>
<td>8.</td>
<td>Frequency comparator VCH-314 Hardware Operational Manual</td>
<td>1</td>
</tr>
<tr>
<td>9.</td>
<td>Frequency comparator VCH-314 Software Operational Manual</td>
<td>1</td>
</tr>
</tbody>
</table>
4 The simplified comparator’s scheme

The simplified scheme of the VCH-314 is given on fig.4.1.

Tested signals
5; 10; 100 MHz

\[ f_y \to f_x \to f_x \to f_y \]

\[ 99.9 \text{ MHz} \]

\[ F_{xy1} \to F_x \to F_{xy2} \]

\[ \text{RS-232C} \]

\[ 220 \text{ V} \]

\[ \text{POWER CONVERTER} \]

\[ \text{INTERFACE} \]

\[ \text{1 PPS UNIT} \]

\[ \text{LO} \]

\[ K=1 \cdot 10^3; 1 \cdot 10^6 \]

\[ \text{FREQUENCY COMPARATOR 1} \]

\[ K=1 \cdot 10^3; 1 \cdot 10^6 \]

\[ \text{FREQUENCY COMPARATOR 2} \]

\[ \text{SIGNALS AND POWER BUS} \]

Fig. 4.1

The comparator contains: two identical modules – FREQUENCY COMPARATOR 1, FREQUENCY COMPARATOR 2, 1 PPS UNIT, INTERFACE and POWER CONVERTER.

We have four input signals – \( f_x \), \( f_y \) at the first frequency comparator and \( f_x \), \( f_y \) at the second comparator.

FREQUENCY COMPARATOR 1, 2 multiplies relative frequency difference of input signals by multiplication factor \( K=1 \cdot 10^3 \) or \( 1 \cdot 10^6 \). Physical passband of the comparator related to phase and frequency fluctuations is (10±3) Hz. Output signals of the comparator \( F_{xy1}, F_{xy2} \) (after multiplication) and reference signal \( F_x \) throw the SIGNALS AND POWER BUS are connected to 1 PPS UNIT, which is dual TIME INTERVALS RECORDER.

For comparators’ output signal frequency we can use relations:
As a reference input (counting frequency) for 1 PPS UNIT we use the signal of the local crystal oscillator (LO) of frequency 99.9 MHz. In addition this device produce three 1 PPS signals \(F_{xy1}, F_{xy2}\) and \(F_x\), to feed external TIME INTERVALS COUNTER. Frequency of signals \(F_{xy1}, F_{xy2}\) we can set both 1 Hz or 100 Hz. Reference 1PPS signal \(F_x\) is made of 99.9 MHz signal by frequency division. When signal of frequency 5 or 10 or 100 MHz is connected to \(f_x\) input, the 99.9 MHz signal is phase looked to \(f_x\) signal, otherwise it is free running. At the TIME INTERVALS RECORDER output each second we have data \(Y_1\) and \(Y_2\) which are averaged on \(L\) samples time moments, corresponding to \(F_{xy1}\) and \(F_{xy2}\) pulses. For more detail understanding see fig. 4.2:

\[
Y_{1i} = \frac{1}{L} \sum_{j=1}^{L} t_{xy1,i,j} 
\]

\[
Y_{2i} = \frac{1}{L} \sum_{j=1}^{L} t_{xy2,i,j} 
\]

\[
\sum_j = \frac{1}{L} \sum_j \frac{f_{xy1} - f_x}{f_x} 
\]

\[
\sum_j = \frac{1}{L} \sum_j \frac{f_{xy2} - f_x}{f_x} 
\]

\[
Y_1 = \frac{1}{L} \sum_{j=1}^{L} t_{xy1,i,j} 
\]

\[
Y_2 = \frac{1}{L} \sum_{j=1}^{L} t_{xy2,i,j} 
\]

Fig. 4.2

Sampling time: \(T=10\) ms. We use averaging to change effective passband \(B_e\) of the comparator.

Frequency response of the comparator, affecting tested signal phase fluctuations spectra (in power units) is close to

\[
W^2(f) = \frac{f_h^2}{f^2 + f_h^2} \frac{\sin^2(L\pi T)}{(L\pi T)^2} 
\]

Where: \(f_h = B/2 = 5\) Hz – physical width of passband.
The Software allows to set \( L=1 \) (no averaging, \( B_c=B=10 \, \text{Hz} \)) and \( L=32 \) (\( B_c=3 \, \text{Hz} \)). \( W^2(f) \) function’s diagram, when \( B=10 \) and 3 Hz (\( L=32 \)) we can see on fig. 4.3.

![Frequency response, \( W^2(f) \)](image)

**Fig. 4.3** The VCH-314 phase fluctuations frequency response.

1 – the VCH-314 real frequency response, \( B=3 \, \text{Hz} \);
2 – first order analogue filter frequency response, \( B=3 \, \text{Hz} \);
3 – the VCH-314 analogue filter frequency response, \( B=10 \, \text{Hz} \).

In this case relative frequency difference \( y_{xy1,i} \) and \( y_{xy2,i} \), measured on \( \tau=M \) seconds can be calculated by the formula (\( M=1, 2, . . . \) – multiplier to change \( \tau \) value):

\[
y_{xy1,i}(\tau) = \frac{1}{K} \left( \frac{\tau}{Y_{1,i,M} - Y_{1,i} + \tau} - 1 \right) \quad (4.6)
\]

\[
y_{xy2,i}(\tau) = \frac{1}{K} \left( \frac{\tau}{Y_{2,i,M} - Y_{2,i} + \tau} - 1 \right) \quad (4.7)
\]

When measurements in different channels were made simultaneously (after special channels’ synchronisation) we have:

\[
y_{y1,y2,i} = y_{xy2,i} - y_{xy1,i} \quad (4.8)
\]

Having \( y_{xy1,i} \), \( y_{xy2,i} \) and \( y_{y1,y2,i} \) values we can use “three-oscillator mode”, specifying frequency instability each of three oscillators \([i]\) (see VCH-314 Software Instruction Manual).

**INTERFACE** is interface of type RS-232 between Comparator and Personal Computer.
5 Measurements performing

5.1 Operating controls and connectors

5.1.1 Operating controls and connectors are located on front and rear panels of the comparator (see fig.5.1 and fig.5.2).

Operating controls of front panel is given in the table 5.1.

<table>
<thead>
<tr>
<th>Position number (fig. 5.1)</th>
<th>Name</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>POWER</td>
<td>Power ON/OFF indicator</td>
</tr>
<tr>
<td>2</td>
<td>Fxy1</td>
<td>Channel XY1 output signal indicator (flashing each 1 second when both of input signals $f_x$ and $f_{y1}$ is present)</td>
</tr>
<tr>
<td>3</td>
<td>Fxy2</td>
<td>Channel XY2 output signal indicator (flashing each 1 second when both of input signals $f_x$ and $f_{y2}$ is present)</td>
</tr>
<tr>
<td>4</td>
<td>Fx</td>
<td>Output reference signal indication (flashing each 1 second)</td>
</tr>
</tbody>
</table>

Fig. 5.1
5.1.2 Rear panel controls and connectors is given in the table 5.2.

**Table 5.2**

<table>
<thead>
<tr>
<th>Position number (fig. 5.2)</th>
<th>Name</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Theta f_x, \Theta f_y$</td>
<td>The second comparator’s inputs</td>
</tr>
<tr>
<td>2</td>
<td>$\Theta f_x, \Theta f_y$</td>
<td>The first comparator’s inputs</td>
</tr>
<tr>
<td>3</td>
<td>$\Theta$1 PPS Fxy1, Fxy2, Fx</td>
<td>1 PPS outputs for external time interval counter</td>
</tr>
<tr>
<td>4</td>
<td>RS-232C</td>
<td>Interface connector</td>
</tr>
<tr>
<td>5</td>
<td>+5 V, +12 V, −12 V</td>
<td>Lamps indicating DC power supply voltages</td>
</tr>
<tr>
<td>6</td>
<td>$\Theta$</td>
<td>Ground connector</td>
</tr>
<tr>
<td>7</td>
<td>220 V, 50 Hz, 30 V·A</td>
<td>AC power connector</td>
</tr>
<tr>
<td>8</td>
<td>F1 AL 250 V</td>
<td>Fuse holders</td>
</tr>
<tr>
<td>9</td>
<td>POWER</td>
<td>power ON/OFF switch</td>
</tr>
</tbody>
</table>
5.2 Signals connecting

5.2.1 Use special cable and connect the RS-232C port of the comparator to COM port of PC.

**Caution!**

To prevent comparator’s interface damage do not connect comparator and computer when comparator is in POWER ON condition.

5.2.2 Plug in power cable and set the POWER switch ON. POWER indicator should light.

5.2.3 Supply input signals to the comparator using one of measurement scheme (see Section 5.4). Warm up the comparator according the table 5.3.

<table>
<thead>
<tr>
<th>Table 5.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired averaging time, ( \tau )</td>
</tr>
<tr>
<td>( \tau \leq 100 \text{ s} )</td>
</tr>
<tr>
<td>( \tau &gt; 100 \text{ s} )</td>
</tr>
</tbody>
</table>

5.2.4 Set measurement parameters according to item 5.3 and Software Operational Manual and start to measure.

5.3 Choosing of the measurement parameters

Before measuring you should set correct measurement parameters which you need:

- multiplication factor – \( K \): (1×10³ or 1×10⁶);
- passband – \( B \): (10 Hz or 3 Hz);
- maximal averaging time – \( T \);
- number of averages – \( N \).

These values determines the following factors:

- maximal measured frequency difference;
- desired measurement resolution and error;
- desired measurement time.

Maximum possible frequency difference (\( y_{\text{max}} \)) depends on the multiplication factor \( K \) and should not exceed 1.0×10⁻⁶ for \( K=1×10^3 \), and 1.0×10⁻⁸ for \( K=1×10^6 \).

Averaging time, \( \tau \) can be set from 1s up to 500000 s.

Passband \( B=10 \text{ kHz} \) for \( K=1×10^3 \) and 10 or 3 Hz for \( K=1×10^6 \).

Measurements frequency resolution (\( y^* \)) is calculated by the formula:

\[
y^* = \frac{10^{-8}}{K\tau}
\]

**Attention!**

For multiplication factor – \( K=1×10^3 \) statistical functions describing the frequency instability for the separate signal (\( \text{var}\{F_x\}, \text{var}\{F_{y1}\}, \text{var}\{F_{y2}\}, \text{var2}\{F_x\}, \text{var2}\{F_{y1}\}, \text{var2}\{F_{y2}\}) \) are calculated incorrectly.
Note
Be careful in the measurement with a great frequency difference. Remember that in this case there is an additional frequency instability \( (\sigma_{\text{ad}}(\tau) \leq 2.0 \times 10^{-3} \Delta f/f) \). Therefore for precise frequency instability measuring it is recommended to set minimal frequency difference.

5.4 Measurement error
There are three types of measurement error in the device:
- frequency instability because of the measurement channels noise (main error, “noise floor”),
- frequency instability due to signals interference (additional error, spurious modulation).
- frequency instability due to outside temperature changing (temperature instability).

5.4.1 “Noise floor”. It is the instability introduced by Comparators’ noise, when \( \Delta f = 0 \). To measure this type of error use Power splitter ZFSC-2-1W+ and special short cables to supply the same signal to reference (\( \Theta fx \)) and to one of tested (\( \Theta fy \)) inputs of the Comparator (see p.5.5.4). At each input signal level should be more than 0.8Vrms. Then you can measure introduced by Comparator’s noise frequency instability.

5.4.2 Spurious modulation. It is the additional instability of the Comparator when \( \Delta f \neq 0 \). In this case due to interference between input signals there is spurious sine-form phase modulation of signals inside the Comparator. We can measure the frequency and the amplitude of spurious phase modulation –\( T_i(F) \) (in units of time). There are three modulation frequencies in the Comparator – \( F = \Delta f, 10\Delta f \) and \( 20\Delta f \). There \( \Delta f = f_x - f_y \) is frequency difference at comparator inputs (in assumption that both signal’s frequency is 5 MHz). When we have specified \( T_i(F) \) values we can calculate introduced additional frequency instability for special \( \Delta f \) and \( \tau \) value using frequency response of calculated frequency instability function. According the specifications we have \( (\sigma_{\text{ad}}(\tau) \leq 2.0 \times 10^{-3} \Delta f/f) \). The only way to reduce this kind of error is to decrease \( \Delta f \).

5.4.3 Temperature instability. Changing of outside temperature causes additional internal phase shift and frequency measurement error. We specify frequency instability of the Comparator less than \( 5.0 \times 10^{-16} \) for measured signals combinations – XY1, XY2 when \( \tau \geq 1000 \) s and outside temperature changing is less than 1 °C/hour.
5.5 Measurements schemes

Attention!
The signal at «Ωfx» input of first comparator must always be connected independently from measurements scheme.

5.5.1 “Two-oscillator, one-channel” mode (Fig. 5.3). It is the simplest traditional mode, when we use only one measuring channel (first comparator – “Ωfx” and “Ωfy” inputs). In this case the results (mathematical expectation) is a total instability of both signals and comparators (we have shifted result). Table 5.4 shows available functions and corresponding mathematical expectation (E{...}), calculated in supposition that frequency fluctuations of all signals and comparator channels are independent.

Table 5.4 Available functions for “two oscillators, one-channel” mode
(see VCH-314 Software Operating Manual)

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculated functions</th>
<th>Mathematical expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean Relative Freq. Diff. (E{F_{y1x}})</td>
<td>(y_{y1}^N - y_{y1}^N + y_{c1}^N)</td>
</tr>
<tr>
<td>2</td>
<td>RMS Relative Freq. Diff. (var{F_{y1x}})</td>
<td>(\sqrt{\delta_{y1}^2 + \delta_{x}^2 + \delta_{c1}^2})</td>
</tr>
<tr>
<td>3</td>
<td>RMS Two-sample Freq. Diff.(var2{ F_{y1x}})</td>
<td>(\sqrt{\sigma_{y1}^2 + \sigma_{x}^2 + \sigma_{c1}^2})</td>
</tr>
</tbody>
</table>

where:

- \(y_{y1}^N, y_{x}^N\) – averaged on total monitoring time \(N \cdot \tau\), relative frequency deviation of \(f_x\) and \(f_{y1}\) signals from the nominal frequency;
- \(y_{c1}^N\) – mean frequency difference, introduced by the comparator 1;
- \(\delta_{y1}, \delta_{x}, \delta_{c1}\) – N-sample Allan variance of \(f_{y1}, f_x\) signals’ and comparator 1;
- \(\sigma_{y1}, \sigma_{x}, \sigma_{c1}\) – Two sample Allan variance of \(f_{y1}, f_x\) signals’ and comparator 1.

Instability of the comparator \(y_{c1}, \delta_{c1}, y_{c1}\) (“noise floor”) we can find, when the same signal is connected to both comparator inputs. If we are looking for the instability of \(f_{y1}\) signal the second and the third terms of mathematical expectations’ represents shift or systematic error due to reference and comparator. When using this scheme we have to be sure that reference signal’s and comparator’s instability is much less than of tested signal.

As a measure of measurement uncertainty or occasional error for two samples Allan variance estimate \(\text{var2} (F_{y1x})\) we can use relative root mean square deviation

\[
d = \sqrt{\frac{E(\sigma_{x1,N}^2)^2 - [E(\sigma_{x1,N}^2)]^2}{[E(\sigma_{y1,N}^2)]^2}}
\] (5.2)
It is shown [2] that

$$d \leq \sqrt{\frac{2}{N}}$$  \hspace{1cm} (5.3)

5.5.2. "Two-oscillators, two-channels mode" (Fig. 5.4). It is the modification of two oscillator's mode when by using two identical measuring channels we can reduce comparator’s measurements error. In this case we supply the same signals \(f_x\) and \(f_y\) to both comparators.

![Fig.5.3 “Two-oscillators, one-channel” mode. Signals connecting](image-url)
Table 5.5 shows mathematical expectation of calculated functions [3] (see p. 3.2 Software Operating Manual). This relations help to estimate first systematic error due to instability of comparators and reference signal. We see that all results are shifted up).

**Table 5.5** Available functions for “two-oscillators, two-channels” mode
(see VCH-314 Software Operating Manual)

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculated functions</th>
<th>Mathematical expectation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean Relative Freq. Diff. (E{ F_{y1x} })</td>
<td>$y_{y1}^N - y_{x}^N + y_{c1}^N$</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>Mean Relative Freq. Diff. (E{ F_{y2x} })</td>
<td>$y_{y2}^N - y_{x}^N + y_{c2}^N$</td>
<td>*</td>
</tr>
<tr>
<td>3</td>
<td>Mean Relative Freq. Diff. (E{ F_{yly2} })</td>
<td>$y_{c1}^N - y_{c2}^N$</td>
<td>**</td>
</tr>
<tr>
<td>4</td>
<td>RMS Relative Freq. Diff. (var{ F_{y1x} })</td>
<td>$\sqrt{\delta_{y1}^2 + \delta_{x}^2 + \delta_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>5</td>
<td>RMS Relative Freq. Diff. (var{ F_{y2x} })</td>
<td>$\sqrt{\delta_{y2}^2 + \delta_{x}^2 + \delta_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>RMS Relative Freq. Diff. (var{F_{yly2}})</td>
<td>$\sqrt{\delta_{c1}^2 + \delta_{c2}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>7</td>
<td>RMS Relative Freq. Diff. (var{F_{x}})</td>
<td>$\sqrt{\delta_{x}^2 + \delta_{y}^2}$</td>
<td>***</td>
</tr>
<tr>
<td>8</td>
<td>RMS Relative Freq. Diff. (var{F_{y1}})</td>
<td>$\sqrt{\delta_{c1}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>9</td>
<td>RMS Relative Freq. Diff. (var{F_{y2}})</td>
<td>$\sqrt{\delta_{c2}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>10</td>
<td>RMS Two-sample Freq. Diff. (var2{F_{y1x}})</td>
<td>$\sqrt{\sigma_{y1}^2 + \sigma_{x}^2 + \sigma_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>11</td>
<td>RMS Two-sample Freq. Diff. (var2{F_{y2x}})</td>
<td>$\sqrt{\sigma_{y2}^2 + \sigma_{x}^2 + \sigma_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>12</td>
<td>RMS Two-sample Freq. Diff. (var2{F_{yly2}})</td>
<td>$\sqrt{\sigma_{c1}^2 + \sigma_{c2}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>13</td>
<td>RMS Two-sample Freq. Diff. (var2{F_{y1}})</td>
<td>$\sqrt{\sigma_{y}^2 + \sigma_{x}^2}$</td>
<td>***</td>
</tr>
<tr>
<td>14</td>
<td>RMS Two-sample Freq. Diff. (var2{F_{y1}})</td>
<td>$\sqrt{\sigma_{c1}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>15</td>
<td>RMS Two-sample Freq. Diff. (var2{F_{y2}})</td>
<td>$\sqrt{\sigma_{c2}^2}$</td>
<td>**</td>
</tr>
</tbody>
</table>

**Notes:**
- * – results, having systematic error due to reference and comparators;
- ** – results, specifying instability of comparators;
- *** – results, having systematic error due to reference only.

For functions number 7 and 13 (cross-variance) we have the second systematic error, related with time delay of measuring in two channels.

Really functions 7 and 13 are autocorrelation estimates which depends on measurement time – $\tau$, frequency fluctuations spectra and time delay between
measuring in different channels – \( \Delta \) (\( \Delta \) is time in seconds of “Time Samples” diagram, curve \( F_yy_1 \), see p. 7.3 Software Operating Manual).

For the two-sample Allan variance \( \sigma_y (\tau) \) (function 13) we have

\[
\text{var} \{ F_x \} = \sqrt{\sigma^2_y(\tau) r^2(\Delta)}, \quad (5.4)
\]

where: \( r (\Delta) \) – auto-correlation function of measured frequency fluctuations.

If we use traditional frequency fluctuations model

\[
S_y (f) = \sum_{\alpha=2}^{2} h_\alpha f^\alpha, \quad (5.5)
\]

**Fig.5.4** “Two-oscillators, two-channels” mode.

Signals connecting

For the two-sample Allan variance \( \sigma_y (\tau) \) (function 13) we have

\[
\text{var} \{ F_x \} = \sqrt{\sigma^2_y(\tau) r^2(\Delta)}, \quad (5.4)
\]

where: \( r (\Delta) \) – auto-correlation function of measured frequency fluctuations.

If we use traditional frequency fluctuations model

\[
S_y (f) = \sum_{\alpha=2}^{2} h_\alpha f^\alpha, \quad (5.5)
\]
In frequency domain we have the relation

\[ r^2(\Delta, \alpha, f_h, M, \tau) = \frac{\int_{0}^{\infty} \alpha f \left( \frac{f_h^2}{f^2 + f_h^2} \right) \sin^2 \left( M \pi f T \right) \sin^4 \left( \pi f \tau \right) \cos \left( 2 \pi f \Delta \right) df}{\int_{0}^{\infty} \alpha f \left( \frac{f_h^2}{f^2 + f_h^2} \right) \sin^2 \left( M \pi f T \right) \sin^4 \left( \pi f \tau \right) df} \]  

(5.6)

Figure 5.5 shows \( r^2(\Delta, \alpha, f_h, M, \tau) \) diagrams for different \( \alpha \) values when \( f_h=5 \) Hz, \( M=32 \), \( T=10 \) ms, \( \tau=1 \) and 10 s (it is measuring condition when \( B=10 \) Hz).

One can see that we have enough measuring accuracy if \( \Delta<0.1 \) s. When we start measuring Software performs channel synchronisation and set \( \Delta<0.0001 \) s. For “two-oscillators, two-channels” measuring mode this value stay near the same in time and there is no need to take care of it.

Occasional error for this case corresponds to the formula (5.3).
5.5.3 “**Three oscillators, two-channel** mode (Fig. 5.6). It is the most advanced mode, when we use for instability measuring three oscillators and two identical measuring channels. One can see three main advantages of this mode:
- simultaneous frequency instability measuring of three oscillator’s
- calculation instability of each individual oscillator
- reduced systematic error due to instability of reference and comparator.

Table 5.6 shows calculated functions and corresponding mathematical expectation [3] (see p. 3.2 Software Operating Manual).

**Table 5.6** Available functions for “three oscillators, two-channels” mode

(see VCH-314 Software Operating Manual)

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculated functions</th>
<th>Mathematical expectation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean Relative Freq. Diff. (E{ F_{y1x} })</td>
<td>$y_{y1}^N - y_{x}^N + y_{c1}^N$</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>Mean Relative Freq. Diff. (E{ F_{y2x} })</td>
<td>$y_{y2}^N - y_{x}^N + y_{c2}^N$</td>
<td>*</td>
</tr>
<tr>
<td>3</td>
<td>Mean Relative Freq. Diff. (E{ F_{y1y2} })</td>
<td>$y_{y2}^N - y_{y1}^N + y_{y1}^N - y_{c2}^N$</td>
<td>*</td>
</tr>
<tr>
<td>4</td>
<td>RMS Relative Freq. Diff. (var{ F_{y1x} })</td>
<td>$\sqrt{\delta_{y1}^2 + \delta_{x}^2 + \delta_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>5</td>
<td>RMS Relative Freq. Diff. (var{ F_{y2x} })</td>
<td>$\sqrt{\delta_{y2}^2 + \delta_{x}^2 + \delta_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>RMS Relative Freq. Diff. (var{ F_{y1y2} })</td>
<td>$\sqrt{\delta_{y1}^2 + \delta_{y2}^2 + \delta_{c1}^2 + \delta_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>7</td>
<td>RMS Relative Freq. Diff. (var{ F_x })</td>
<td>$\sqrt{\delta_{x}^2}$</td>
<td>***</td>
</tr>
<tr>
<td>8</td>
<td>RMS Relative Freq. Diff. (var{ F_{y1} })</td>
<td>$\sqrt{\delta_{y1}^2 + \delta_{c1}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>9</td>
<td>RMS Relative Freq. Diff. (var{ F_{y2} })</td>
<td>$\sqrt{\delta_{y2}^2 + \delta_{c2}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>10</td>
<td>RMS Two-sample Freq. Diff.(var2{ F_{y1x} })</td>
<td>$\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2 + \sigma_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>11</td>
<td>RMS Two-sample Freq. Diff.(var2{ F_{y2x} })</td>
<td>$\sqrt{\sigma_{y2}^2 + \sigma_{x}^2 + \sigma_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>12</td>
<td>RMS Two-sample Freq. Diff.(var2{ F_{y1y2} })</td>
<td>$\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2 + \sigma_{c1}^2 + \sigma_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>13</td>
<td>RMS Two-sample Freq. Diff.(var2{ F_x })</td>
<td>$\sqrt{\sigma_{x}^2}$</td>
<td>***</td>
</tr>
<tr>
<td>14</td>
<td>RMS Two-sample Freq. Diff.(var2{ F_{y1} })</td>
<td>$\sqrt{\sigma_{y1}^2 + \sigma_{c1}^2}$</td>
<td>**</td>
</tr>
<tr>
<td>15</td>
<td>RMS Two-sample Freq. Diff.(var2{ F_{y2} })</td>
<td>$\sqrt{\sigma_{y2}^2 + \sigma_{c2}^2}$</td>
<td>**</td>
</tr>
</tbody>
</table>

**Notes:** * – results, having systematic error due to instability of reference and comparators;
** – results, having systematic error due to instability of comparators;

*** – results, having no systematic error.

One can see that only “X” signal we can measure without systematic error.

In this case for functions 7,8,9, 13,14,15 there is the second systematic error, caused by time delay of measuring in channels ($\Delta$). To have appropriate measurements precision you need to find maximal allowable $\Delta^*$ value by using relation (5.6) and figures 5.4, 5.5 and to be sure that while measuring $\Delta \leq \Delta^*$. But in this case user should be careful because delay $\Delta$ is growing in time after measuring start

$$\Delta = |\Delta_o + 10^K y_{y1y2} t|,$$  \hspace{2cm} (5.7)

where: $\Delta_o < 0.0001$ s – initial $\Delta$ value;

$y_{y1y2}$ – relative frequency difference of $Y_1$ and $Y_2$ signals;

$K$ – multiplication factor, $t$ – time elapsed from measuring start.

If it is impossible to guarantee $\Delta \leq \Delta^*$ condition while measuring you may use only part of data and “Recalc” function (see Section 6.7 Software Operating Manual).

Fig.5.6 “Three oscillators two-channels” mode.

Signals connecting.
To estimate occasional error for \text{var2}\{Fx\}, \text{var2}\{Fy1\}) and \text{var2}\{Fy2\}) we can use the following formulas\cite{4}.

\[
\begin{align*}
        d_x & \leq \frac{A}{N} \left( 2 + \frac{\sigma_{y1}^2 + \sigma_{c1}^2}{\sigma_x^2} + \frac{\sigma_{y2}^2 + \sigma_{c2}^2}{\sigma_x^2} + \frac{\sigma_{y1}^2 + \sigma_{c1}^2}{\sigma_y^2} \right), \\
        d_{y1} & \leq \frac{A}{N} \left( 2 + \frac{\sigma_{x1}^2 + \sigma_{c1}^2}{\sigma_{y1}^2} + \frac{\sigma_{x2}^2 + \sigma_{c2}^2}{\sigma_{y1}^2} + \frac{\sigma_{x1}^2 + \sigma_{c1}^2}{\sigma_{y2}^2} \right), \\
        d_{y2} & \leq \frac{A}{N} \left( 2 + \frac{\sigma_{x1}^2 + \sigma_{c1}^2}{\sigma_{y1}^2} + \frac{\sigma_{x2}^2 + \sigma_{c2}^2}{\sigma_{y1}^2} + \frac{\sigma_{x1}^2 + \sigma_{c1}^2 + \sigma_{c2}^2}{\sigma_{y2}^2} \right)
\end{align*}
\]  

(5.8)

where: \( A \) – depends on correlational features of tested signals, \((1 \leq A < 2)\)

Occasional error formulas help to find minimal \( N \)-value to have appropriate measurements accuracy.

Notes:

All error calculations where made supposing that frequency fluctuations of tested signals’ and comparators’ are independent. It is close to real practice only for short-term stability \((\tau = 1; 10\) and may be \(100\) s).
5.5.4 *The main error measuring* ("noise floor" when $\Delta f/f=0$). Connect signals according Fig. 5.7 and set measuring parameters according Fig. 5.8.

After waiting for 4 hours initiate measuring. Table 5.7 shows calculated functions’ mathematical expectation.

**Fig. 5.7** Signals connecting to measure the main instability of the comparator ("noise floor" when $\Delta f/f=0$)

The comparator is perfect if measured statistical functions describing the frequency instability are not more values given in the table 2.1
Table 5.7 The mathematical expectation of calculated functions’ when comparators “noise floor” measuring.
(see VCH-314 Software Operating Manual)

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculated functions</th>
<th>Mathematical expectation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean Relative Freq. Diff. (E{ F_{y1x} })</td>
<td>$y_{c1}^N$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mean Relative Freq. Diff. (E{ F_{y2x} })</td>
<td>$y_{c2}^N$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Mean Relative Freq. Diff. (E{ F_{y1y2} })</td>
<td>$y_{c1}^N - y_{c2}^N$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>RMS Relative Freq. Diff. (var{ F_{y1x} })</td>
<td>$\sqrt{\delta_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>5</td>
<td>RMS Relative Freq. Diff. (var{ F_{y2x} })</td>
<td>$\sqrt{\delta_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>RMS Relative Freq. Diff. (var{ F_{y1y2} })</td>
<td>$\sqrt{\delta_{c1}^2 + \delta_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>7</td>
<td>RMS Relative Freq. Diff. (var{ F_x })</td>
<td>0</td>
<td>**</td>
</tr>
<tr>
<td>8</td>
<td>RMS Relative Freq. Diff. (var{ F_{y1} })</td>
<td>$\sqrt{\delta_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>9</td>
<td>RMS Relative Freq. Diff. (var{ F_{y2} })</td>
<td>$\sqrt{\delta_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>10</td>
<td>RMS Two-sample Freq. Diff. (var2{ F_{y1x} })</td>
<td>$\sqrt{\sigma_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>11</td>
<td>RMS Two-sample Freq. Diff. (var2{ F_{y2x} })</td>
<td>$\sqrt{\sigma_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>12</td>
<td>RMS Two-sample Freq. Diff. (var2{ F_{y1y2} })</td>
<td>$\sqrt{\sigma_{c1}^2 + \sigma_{c2}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>13</td>
<td>RMS Two-sample Freq. Diff. (var2{ F_x })</td>
<td>0</td>
<td>**</td>
</tr>
<tr>
<td>14</td>
<td>RMS Two-sample Freq. Diff. (var2{ F_{y1} })</td>
<td>$\sqrt{\sigma_{c1}^2}$</td>
<td>*</td>
</tr>
<tr>
<td>15</td>
<td>RMS Two-sample Freq. Diff. (var2{ F_{y2} })</td>
<td>$\sqrt{\sigma_{c2}^2}$</td>
<td>*</td>
</tr>
</tbody>
</table>

Notes: * – results, having systematic error due to instability of comparators; ** – results, having no systematic error.
One can see that only “x” signal we can measure without systematic error.
5.5.5 Frequency comparators’ additional error measuring (when $\Delta f/f \neq 0$). Connect signals of 5 MHz frequency according Fig. 5.4. At least one signal X or Y should be capable to change frequency. Set measured relative frequency difference in the range $(5.1 \div 5.5) \times 10^{-9}$.

Set parameters of measuring according Fig. 5.9, and start measuring program. When measuring is finished find Allan variance values for 1-second average time ($\text{var}2(\text{Fy1x})$, $\text{var}2(\text{Fy2x})$) and relative frequency difference ($E(\text{Fy1x})$, $E(\text{Fy2x})$).

The comparator is perfect if: $\text{var2} \{\text{Fy1x}\} < 0.002 \times E(\text{Fy1x})$;
$\text{var2} \{\text{Fy2x}\} < 0.002 \times E(\text{Fy2x})$. 

Fig. 5.8 Parameters list for the main instability measuring
Fig. 5.9 Parameters list for additional instability measuring

References:
3. И.Н. ЧЕРНЫШЕВ. АППАРАТУРНАЯ СОСТАВЛЯЮЩАЯ СИСТЕМАТИЧЕСКОЙ ПОГРЕШНОСТИ ИЗМЕРЕНИЯ НЕСТАБИЛЬНОСТИ ЧАСТОТЫ МЕТОДОМ ТРЕХ ГЕНЕРАТОРОВ. ТЕХНИКА СРЕДСТВ СВЯЗИ. Серия РИТ. 1990, выпуск 2.
4. Г. П. ПАШЕВ, И. Н. ЧЕРНЫШЕВ. ПОГРЕШНОСТЬ ИЗМЕРЕНИЯ НЕСТАБИЛЬНОСТИ ЧАСТОТЫ МЕТОДОМ ТРЕХ ГЕНЕРАТОРОВ. ТЕХНИКА СРЕДСТВ СВЯЗИ. Серия РИТ. 1987, выпуск 6.